## THE RECOVERED EMPIRE

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ABSTRACT. An abacus, drawn by Felipe Guaman Poma de Ayala (fig. 0), allows us to discover the Inca astronomical reckoning which is founded on a mixed base 36-40 numeral system, built on numbers belonging to Fibonacci sequence. Such an eccentric base, confirmed by many archaeological findings, reveals considerable astronomical knowledge, like Venus and Mercury's cycles. The same abacus shows the Inca sidereal solar year, with its upsetting accuracy, and a wonderful perpetual calendar, surprisingly based on number 5. Some application of the Inca system, like the Atahualpa calculator, is possible even nowadays.


Figure 0

Mauro- Hello Uncle! I bought this fine book for you: The Crest of the Peacock (fig. 1a).
Nicolino- It is certainly a book on Mathematics; an Indian saying states: "Like the crest of a peacock, so is Mathematics at the head of all knowledge". Let us see the cover! Oh!.... Like in Fibonacci sequence: each term is the sum of the two previous terms $(\mathbf{3}=\mathbf{2}+\mathbf{1}$ and $\mathbf{5 = 3 + 2})$.


Figure 1
Opening the book at page $53 \ldots$ (fig.1b)
$\mathbf{N}$ - This Inca abacus has five rows and four columns: in each row we can see four frames in which it is possible, according to the drawn circles, to separately put $\mathbf{1 , 2 , 3}$ or $\mathbf{5}$ seeds. The seeds should not have the same value, otherwise we would have only one square per row. After that we'll proceed by giving different weights to the seeds, the same weights suggested by the circles: 1, 2, $\mathbf{3}$ or $\mathbf{5}$. That means these numbers of Fibonacci sequence are given a positional value.

U


Figure 2

M- So, starting from the right, I can put one seed, weighing 1, at the column of the units $\mathbf{U}$ (fig 2a); then I place two seeds, both weighing 2, at the column of the couples $\mathbf{C}$, reaching here the value 4 (fig 2b); I put now three weighing $\mathbf{3}$ seeds in the column of the triples $\mathbf{T}$, obtaining here the value $\mathbf{9}$ (fig 2 c ). Lastly I'm going to place five seeds, each one weighing $\mathbf{5}$, at the column of the quintets $\mathbf{Q}$, thus reaching the value $\mathbf{2 5}$ (fig 2d). Adding the values of all the frames (fig 3 ) I obtain $\mathbf{3 9}(\mathbf{1 + 4 + 9}$ $+25=39$ )


Figure 3
$\mathbf{N}$ - Very well! Let's perform some simple addictions!
M- Like $\mathbf{8 + 7 = 1 5}$, with $\mathbf{8 = 5 + 3}$ (fig. 4a) and $7=5+2$ (fig. 4b). By simply adding in each square Q, T, C I obtain $10+3+2$ (fig. 4 c ) and, substituting the seeds with weight 3 and 2 with one weighing 5 seed, I can read $\mathbf{1 5}$ (fig. 4d). It's amusing!


Figure 4
$\mathbf{N}$ - Let's perform another addiction!


Figure 5

M- Yes! $\mathbf{1 7 + 9 = 2 6}$ is good for me. I make up $\mathbf{1 7}$ by $\mathbf{1 5 + 2}$ (fig. 5 a ) and 9 by $\mathbf{5 + 3 + 1}$ (fig. 5 b), so I can easily obtain $20+3+2+1$ (fig. 5c) and lastly $\mathbf{2 5 + 1 = 2 6}$, just with a Quintet seed in place of a Triple and a Couple seed. Wonderful! ... But I cannot completely understand this numeral system ... No problem up to 39 ...
$\mathbf{N}$ - When considering the absence of seeds too we have $\mathbf{4 0}$ digits which involve a base $\mathbf{4 0}$ numeral system.

M- Why such a strange system?
$\mathbf{N}$ - You must keep in mind that Incas used to live in perfect harmony with nature. Let's think about the thousands of fruit variety which they have been selecting through the centuries, like tomatoes, peppers, beans, potatoes, They even derived the reckoning system from nature!

M- What? Are you sure?


Figure 6
N - Certainly, dear! Let us consider $\mathbf{4 0}=\mathbf{1}^{\mathbf{2}}+\mathbf{1}^{\mathbf{2}}+\mathbf{2}^{\mathbf{2}}+\mathbf{3}^{\mathbf{2}}+\mathbf{5}^{\mathbf{2}}$ and draw the squares with side length $\mathbf{1 , 2}, \mathbf{3}$ and $\mathbf{5}$ like in figure 6 a: we can easily obtain an Andean spiral which is similar to the nautilus one (fig. 6b). This spiral, which was constantly appreciated by Inca and preinca cultures, is painted on hundreds of ceramics, especially on those belonging to the Moche environment (fig. 7).


Figure 7

M- I still don't understand the usefulness of a base 40. You can definitely recognize that it is highly eccentric!
$\mathbf{N}$ - Not so much as the Mayan one.
M- Oh! I'm getting there .... Incas used this base $\mathbf{4 0}$ for astronomical reckoning! And, if Dresda code suggests us that Mayans used a mixed base 18-20, I can imagine that Incas had a proportional base 36-40. Isn't it?
$\mathbf{N}$ - Bully for you! Actually if the carry occurs at $\mathbf{3 6}$, only in the first row, and at 40, in the left rows, we can open a window, on the wonderful Inca culture, which reveals a considerable astronomical knowledge. Such mixed base allows us to easily find hundreds of archaeological findings which prove the correctness of the base itself.

M- Well! How can I obtain this fantastic base 36-40?
$\mathbf{N}$ - As if by a spell ... Let's leave out the frame of couples $\mathbf{C}$, only at the first row!
M- So I have a carrying oversaturation if I add one seed with weight $\mathbf{1}$ when all other frames are full: $5 \times 5+\mathbf{3} \times 3+\mathbf{1}+\mathbf{1}=\mathbf{3 6}$ (fig 8a); while, in other rows, I have a carrying regular oversaturation if I add $\mathbf{1}$ when also the frame of couples $C$ is full: $\mathbf{5} \times \mathbf{5}+\mathbf{3 \times 3 + 2 \times 2 + 1 + 1 = 4 0}$ (fig. 8b).


Figure 8
N- We may not realize it, but now we are able to fill Guaman Poma abacus and to total each row and all the rows.

|  |  | Q $\mathbf{T}$ |  | C | U |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 92,159,999 | 89,856,000 | 5*11,520,000 | 3*6,912,000 | $2 * 4,608,000$ | 2,304,000 | $36 * 40^{3}$ |
| 2,303,999 | 2,246,400 | 5*288,000 | 3*172,800 | 2*115,200 | 57,600 | $36 * 40^{2}$ |
| 57,599 | 56,160 | 5*7,200 | 3*4,320 | 2*2,880 | 1,440 | $36 * 40^{1}$ |
| 1,439 | 1,404 | 5*180 | 3*108 | 2*72 | 36 | $36^{1}$ |
|  | 35 | 5*5 | 3*3 |  | 1 | $40^{0}$ |

Figure 9

M- I already know the total of the first row: 35. So the second row starts with $\mathbf{3 6}$ (fig. 9) in the column of units U, $\mathbf{2 \times 7 2}$ follow in the couples $C, \mathbf{3 \times 1 0 8}$ in the triples $\mathbf{T}$ and $\mathbf{5 \times 1 8 0}$ in the column of quintets $Q$. Therefore I have the total of second row $1404(36 \times 39=1404)$ and the general total $1439(1404+\mathbf{3 5}=\mathbf{1 4 3 9})$. The third row will start with 1440 and so on!
Wahoo! Every number, from $\mathbf{1}$ to $\mathbf{9 2 , 1 5 9 , 9 9 9}$, can be properly symbolized by simple seeds!
$\mathbf{N}$ - We must curb our enthusiasm. They are just cold numbers, without astronomical pregnancy ... It's time to speak of the harmonic year.

M- What is a harmonic year?
$\mathbf{N}$ - The lunar year, with $\mathbf{1 2}$ moons and $\mathbf{3 5 4 . 3 6 7 0 8}$ days, and the solar one, with $\mathbf{1 2}$ months and 365.25636 days, are well mediated by a year with $\mathbf{3 6 0}$ days, which we can define harmonic year.

M- Why?
N - Let's consider the identity $5^{1} \times \mathbf{3}^{2} \times \mathbf{2}^{\mathbf{3}} \times \mathbf{1}^{5}=5 \times 9 \times 8 \times 1=\mathbf{3 6 0}$.
M- What a spectacular base-exponent combination with the first terms of Fibonacci sequence! I can see a strong criterion of fairness because smaller exponents are coupled to larger bases and vice versa. I suppose that the harmonic year was very important for Incas.
$\mathbf{N}$ - Not only for them; for Nuragics, Egyptians and Mayans too!

|  | Q | T | C | U |
| :---: | :---: | :---: | :---: | :---: |
| harmonic years | $5 * 32,000$ | $3 * 19,200$ | $2 * 12,800$ | 6,400 |
| harmonic years | $5 * 800$ | $3 * 480$ | $2 * 320$ | 160 |
| harmonic years | $5 * 20$ | $3 * 12$ | $2 * 8$ | 4 |
| days | $5 * 180$ | $3 * 108$ | $2 * 72$ | 36 |
| days | $5 * 5$ | $3 * 3$ |  | 1 |
|  |  |  |  |  |

Figure 10
M- In fact things become much easier if I consider that the first two rows are representative of days, while those ones that are left are for harmonic years (fig. 10). An example: in the column $\mathbf{U}$ of the third row, $\mathbf{4}$ years $(\mathbf{4} \times 360=1440)$ have to be related to the leap year.
$\mathbf{N}$ - It was just that! It's early for such a consideration.. First of all let's perform two simple addictions and see how to handle the carry; in such a way we are able to freely move from a level to another. After that we shall try to find some historical or archaeological confirmation.

M- Well, I'm ready! I do want to make two addictions, one on the first row and another on the
 and 12 ). I can see that the carry always occurs when there are eight seeds: more precisely six quintet seeds and two triple seeds (fig 11c) on the first row ( $\mathbf{6 \times 5 + 2 \times 3 = 3 6}$ ) and eight quintet seeds (fig. 12 c ) on the second row ( $\mathbf{8} \times \mathbf{5}=\mathbf{4 0}$ ). So number eight is strategically important in carrying management.


Figure 11
Q

| $\bigcirc \bigcirc$ | $\bigcirc$ | $\begin{aligned} & 0 \\ & 0 \end{aligned}$ | $\bigcirc$ |
| :---: | :---: | :---: | :---: |
| $\begin{array}{ll}0 & \\ 0 & 0 \\ 0 & 0\end{array}$ | $\bigcirc$ | $\begin{aligned} & 0 \\ & 0 \end{aligned}$ | $\bigcirc$ |

720
Q

720


1440

Figure 12
$\mathbf{N}$ - Don't forget that 8 belongs to the same Fibonacci sequence (1, 1, $\mathbf{2}, \mathbf{3}, \mathbf{5}, \underline{\mathbf{8}}, \mathbf{1 3}, \mathbf{2 1}, \ldots$.

M- Well! Then there must be some trace of this number in Spanish chronicles.
$\mathbf{N}$ - You are right! José de Acosta is crucial in this regard. He says: "Seeing them use another kind of quipus, with kernels of maize corn, is perfect joy. In order to perform very difficult calculi, for which an able accountant would need paper and pen, these Indians will make use of their kernels. They will put one here, three over there, eight I do not know where. They will move one from here, exchange three over there and they indeed make on time their calculus, without missing a tilde". The same author adds: "Where they apply they are far ahead of us".

M- I had better look for some ancient yupanas, with eight quintets!
$\mathbf{N}$ - It's enough for today! We just have to email the top expert on Incas in Italy, Antonio Aimi, and an electronic engineer genius, my friend Maurizio Orlando. We will see tomorrow!

M- Good bye, Uncle!
The day after ...
$\mathbf{N}$ - We are in luck. Aimi sent me several images of spectacular yupanas: all of them confirm our carrying model.

M- Wonderful! It's important that this model works well on other yupanas, in order to observe the principle of substitutability, which must characterize any deciphering system.


Figure 13
$\mathbf{N}$ - Right! Let's take a look at these images: there are three yupanas which are typified by two levels with eight quintet frames $\mathbf{Q}(\mathbf{8} \mathbf{x}=\mathbf{4 0})$, at the second level (fig 13 a ), and six yupanas which have three levels with two score frames $\mathbf{S}(\mathbf{2} \mathbf{x} \mathbf{2 0}=\mathbf{4 0}$ !) at the third level (fig. 13b). There are also six
images of more ancient yupanas which work with base $\mathbf{1 0}$ but their study is not now within our purpose.

M- Now I'd like to know the astronomical value of black seeds in Guaman Poma yupana; according to common sense these seeds have a strategic importance.


| 180 | 108 | 72 |  |
| :---: | :---: | :---: | :---: |
| 10 | 9 |  | 1 |

Figure 14
$\mathbf{N}$ - These seeds are really more precious than black pearls! In the first row we have $\mathbf{1 , 9}$ and $\mathbf{1 0}$ totalling 20, while in the second we find 72, 108, 180 totalling 360 (fig. 14).

M- The harmonic year!


Figure 15
$\mathbf{N}$ - What an intelligent nephew! But the Author is more detailed, so we have to visit the Guaman Poma website which is one of the most beautiful sites all over the Internet; the great Rolena Adorno acted as scholarly consultant to this magnificent project, highly considering the work of all the scholars. We can read, at page 72:"Contauan los domingos dies dias y un año y los mezes de la luna treynta dias" (fig. 15). Actually 1 Sun-day and 9 workdays were scanning the Inca ten-day week, according to the same author (at page 235, where we can read: "Contauan la semana dies dias" (fig. $16)$ ), so as twelve lunar months, each with $\mathbf{3 0}$ days, perfectly define a harmonic year with its $\mathbf{3 6 0}$ days.


Figure 16
M- But what is the meaning of $\mathbf{2 0}$ and $\mathbf{7 2}$ ?
$\mathbf{N}$ - The vigesimal rhythm, which also characterizes the Mesoamerican cultures, is very useful in computing Venus' cycles, because 20 is sub multiple of 2920 which represents, in days, five Venus' cycles, each one consisting of $\mathbf{5 8 4}$ days $(\mathbf{5 8 4} \mathbf{x} \mathbf{5}=\mathbf{2 9 2 0})$. So number $\mathbf{2 0}$ stays for a venusian month.

M- By the way! The same cycle with 2920 days is findable in the Mayan Dresda code in form of $8 \times 360+2 \times 20$ (fig. 17).
$8 \times 360$
$2 \times 20$


Figure 17
$\mathbf{N}$ - Yes dear! 72 is even more surprising, because Venus reaches its maximum in brightness twice in each cycle, more exactly $\mathbf{3 6}$ days before and $\mathbf{3 6}$ days after the inferior conjunction, just $\mathbf{7 2}$ days away!

M- I suppose that the determination of maximum brightness is very difficult and, if Andean civilizations knew this sophisticated astronomical concept, then I'd like to see some finding which prove it.
$\mathbf{N}-$ You are perfectly right! We know, dear, that among scholars Tom Zuidema richly deserves a special respect. Well! About thirty years ago he classified the Huari canvas at the Staatliches Museum für Volkerkunde of Monaco (Bavaria) as a calendar. Take a look at this reconstruction of the precious canvas (fig. 18)!


Figure 18
M- Wonderful! It has two identical halves, each one is a matrix with ten rows and thirty six columns, with a total of $\mathbf{3 6 0}$ colored circles. The venusian character is crystal clear because, if you know the date of the first Venus' maximum brightness, it's enough to vertically move over the next row ( $\mathbf{3 6}$ days) to find the date of the inferior conjunction; a second similar moving over ( $\mathbf{3 6}$ days) leads to the date of the second Venus' maximum brightness! So two rows of a matrix contain 72
circles, three rows $\mathbf{1 0 8}$ circles and five rows contain $\mathbf{1 8 0}$ circles: $\mathbf{7 2}, \mathbf{1 0 8}, \mathbf{1 8 0}$ and $\mathbf{3 6 0} \ldots$ The same numbers of Guaman Poma yupana! Any randomness has to be resolutely excluded! But why two halves?

N- It's a two-year calendar. We will examine it better later. Did you notice that Guaman Poma put his yupana at page $\mathbf{3 6 0}$ of his manuscript (fig. 19)?


Figure 19
M- To surely highlight the extreme importance of $\mathbf{3 6 0}$ : the harmonic year. But I see another number ... It seems written by pencil ... Why?
$\mathbf{N}$ - Someone renumbered the pages, without considering that Guaman Poma deliberately used this number on a wrong page. It's our typical violence! But let's go on!

M- On the third row I can read $\mathbf{2 8 8 0}$ days which are equivalent to $\mathbf{8}$ harmonic years (fig. 20). Why not $\mathbf{2 9 2 0}$ days, like the Mayan Venus' cycle?


360 |  |  | 2880 |  |
| :---: | :---: | :---: | :---: |
| 180 | 108 | 72 |  |
| 10 | 9 |  | 1 |

Figure 20
N-Let's consider $2920=\mathbf{2 8 8 0}+\mathbf{3 6}+\mathbf{2}+\mathbf{2}$. Guaman Poma was forced to represent $\mathbf{2 8 8 0}$ because 2920 would have altered the deep meaning of the first two rows (fig. 21).

M- In fact we'd have obtained $\mathbf{2 4}(20+4)$ in the first row and $\mathbf{3 9 6}(\mathbf{3 6 0}+\mathbf{3 6})$ in the second, both $\mathbf{2 4}$ and $\mathbf{3 9 6}$ without astronomical value. Since 2880 days are equivalent to $\mathbf{8}$ harmonic years ( $\mathbf{8 \times 3 6 0}=$ 2880), does any finding exist which can work like an eight-year calendar?


Figure 21
$\mathbf{N}$ - It is a very delicate matter, so it is better to search for some canvas with this characteristic at the Balzarotti collection in Milan. Now we are forced to stop. We will see tomorrow!

M- Bye, bye!
The third day, still being in luck ...
$\mathbf{N}$ - I found this remarkable image (fig. 22) in a book that Aimi gave me. What do you think of this Nazca canvas?


Figure 22

M- It looks like a singular chessboard with $\mathbf{2 4}$ rows and $\mathbf{1 5}$ columns, totalling $\mathbf{3 6 0}$ squares .. Interesting ... What about these staircase like decorations (fig 23), with approached in contrast black and white?


Figure 23
$\mathbf{N}$ - They provide $\mathbf{8}$ day marks in any square.
M- So we have $\mathbf{2 8 8 0}$ days ( $\mathbf{3 6 0} \mathbf{x} \mathbf{8}=\mathbf{2 8 8 0}$ ), the same number on the third row of Guaman Poma yupana. What a remarkable perfectly planned coincidence!
$\mathbf{N}$-You are right. This canvas is an eight-year venusian calendar; after four years we need to turn it upside down (fig. 23). Don't we notice a likeness between this canvas and Tom Zuidema calendar?

M- Yes Uncle! They both need to compute apart 5 days (like epagomenal Egyptian days or Mayan Wayeb) to obtain the vague year which is built on $\mathbf{3 6 5}$ days. So adding 40 days in $\mathbf{8}$ years ( $\mathbf{5} \times 8=$ 40) we have $\mathbf{2 9 2 0}$ days like the Mayan cycle! That's why Guaman Poma considered $\mathbf{2 8 8 0}$ days!
$\mathbf{N}$ - This structuring of the canvas (fig. 22) on number 15, multiple of 5, also makes reading easy in:

- 10-day weeks and Sun days ( $2 / 3$ of row),
- 20-day venusian months ( $\mathbf{1}+\mathbf{1 / 3}$ of row),
- 30-day lunar months ( $\mathbf{2}$ rows).

M- But what happens in $\mathbf{2 9 2 0}$ days? I am curious to know it.


Figure 24
$\mathbf{N}$ - In such a period Venus completes five cycles ( $\mathbf{5 8 4} \mathbf{x} \mathbf{5}=\mathbf{2 9 2 0})$ getting to approximately the same position on the ecliptic. We can use a very interesting astronomy software to prove it. On June 8, 2004 Venus crossed the solar disk (fig. 24a); 2920 days after, on June 6, 2012, Venus will cross it again (fig. 24b).
$\mathbf{M}$ - This is a respectable astronomical knowledge! But what is the meaning of the remaining five seeds, on the third row, which are in equivalence with $\mathbf{1 0 0}(\mathbf{2 0} \times \mathbf{5}=\mathbf{1 0 0})$ harmonic years?
$\mathbf{N}$ - It's better to skip over them now; they will be subject for discussion later. Let's move on the fourth row!


| 800 | $\mathbf{4 8 0}$ |  | 160 |
| :---: | :---: | :---: | :---: |
|  | Pachacuti |  |  |
|  | Sol |  |  |
|  |  |  |  |
|  |  |  |  |

Figure 25
M- First I meet $\mathbf{1 6 0}$ years in the column of units (fig. 25). Is this period astronomically significant?
$\mathbf{N}$ - It is the time that separates two spectacular consecutive conjunctions between Mercury and Venus. We can verify this by using again this astronomy software; if we put the date December 25, 240 BC (according to precise indications, hidden by Guaman Poma in the chapter of the Incas) we'll observe a Mercury-Venus conjunction of rare beauty (fig. 26a).


Figure 26

M- 160 years later on November 17, 80 BC There will be another conjunction between Mercury and Venus (fig 26b)! What's the role of Mercury in Inca culture? I suppose it was very important.


Figure 27
$\mathbf{N}$ - You are right! After the Sun and the Moon it was the most important celestial body. According to Guaman Poma Chuqui Ylla (Mercury) was one of the Incas' own arms (fig. 27). The identity Chuqui Ylla-Mercury is suggested by the same author when states, at page 265, that Chuqui Ylla and Chasca Cuyllor (Venus) are sons of Sun and Moon: "...Inti, ... Quilla, y sus hijos Chuqui Ylla, Chasca Cuyllor" (fig. 28); the lunisolar aspect belongs to planets which are characterized by phases. So we are forced to assume the equivalence Chuqui Ylla-Mercury, while the identity Chasca Cuyllor-Venus is unanimously accepted.


Figure 28
M- What about the cycle which is represented by the second seed (fig. 25) whose value is $\mathbf{4 8 0}$ years?

N- Bolivian and Peruvian natives consider a 500 -year cycle that is named Pachacuti: they are wrong because one Pachacuti lasts for $\mathbf{4 8 0}$ years! This is confirmed by many artefacts like the Gateway of the Sun in Tiwanaku.

M- The first seed plus the third (fig. 25) give us $\mathbf{9 6 0}$ years $(\mathbf{8 0 0}+\mathbf{1 6 0}=\mathbf{9 6 0})$. So a Sol cycle is made up of $\mathbf{9 6 0}$ years, that is two Pachacuti. Isn't it?
$\mathbf{N}$ - That's it! Let's go on. Here is the last row!


| $\mathbf{6 4 . 0 0 0}$ |  | $\mathbf{2 5 . 6 0 0}$ | (6.400) |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Figure 29
M- I can recognize only two seeds with their $\mathbf{2 5 , 6 0 0}$ years (fig. 29): the precession of the equinoxes!
$\mathbf{N}$ - Don't worry, it's more than enough! Two 32,000-year cycles are not analysable because our astronomy software ranges over $\mathbf{1 5 , 0 0 0}$ years, from $\mathbf{5 , 0 0 0} \mathrm{BC}$ to $\mathbf{1 0 , 0 0 0} \mathrm{AD}$. We can just suppose that they refers to the Southern Cross. So let's go on deciphering the left seeds.


Figure 30

M- I'd like to know the role of the five seeds in the third row (fig. 30). Could they indicate something more than our century?
$\mathbf{N}$ - Yes! Five weighing $\mathbf{5}$ seeds suggest $\mathbf{2 5}$ corrective days on $\mathbf{1 0 0}$-year periods. In such a way the leap years are considered by inserting 5 days each $\mathbf{2 0}$-year cycle.

M- This means to have a year which is perfectly identical with the Julian one. In fact: $(20 \times 365+5) / 20=365.25$.


Figure 31
$\mathbf{N}$ - We can understand now a very obscure passage of Guaman Poma; referring to Inca months he says, at page 260: "... llegavan a treynta dias o treynta un dia o dos, conforme al menguante" (fig. 31). Now a 30-day month is clear; a 31-day month is intelligible because it leads to the vague year: ( $\mathbf{7} \times 30+5 \times 31=365)$..

M- But, on the first impact, a 32-day month honestly upsets me.
$\mathbf{N}$ - Not at all! Guaman Poma forces us to use our head: if $\mathbf{5}$ epagomenal days are in correspondence with five 31-day months, then further $\mathbf{5}$ days, inserted on the twentieth year (which has $\mathbf{3 7 0}$ days), involve 32-day months:
$(7 \times 30+5 \times 32=370)$ !
M- The Incas well knew the bissextile character of the years!
$\mathbf{N}$ - Not only this! On the fourth row, the $\mathbf{8 0 0}$-year cycle (fig. 30) needs corrective $\mathbf{5}$ days (in fact we have one weighing 5 seed); so the length of the year is modified in $\mathbf{3 6 5 . 2 5 6 2 5}$ :
$(800 \times 365.25+5) / 800=365.25625$.
M- How can I call such a year?
N- I think "septile year" could be good: it corresponds to have five 367-day years.
M- Can I conclude with the fifth row?
$\mathbf{N}$ - Certainly, dear!
M- The 44,800-year cycle (fig. 30) requires further corrective $\mathbf{5}$ days, because we have seeds with both weights $\mathbf{3}$ and $\mathbf{2}(\mathbf{3 + 2 = 5})$. This final correction gives:
$(44,800 \times 365.25625+5) / 44,800=365.2563616$.
... It's shocking! ... It's the sidereal solar year!
$\mathbf{N}$ - It's a bit more accurate than ours. In fact we read on our encyclopaedias a down to the hundred thousandths value, $\mathbf{3 6 5 . 2 5 6 3 6}$ for the same year.

M- Can we exclude any randomness from our reconstruction?


Figure 32
N- Absolutely! It has to be excluded! Guaman Poma was aware of the extreme astronomical accuracy of his people. At pages 883-884 he says: "Juan Yumpa del pueblo de Uchuc Marca, Lucana, ... astrologo pueta $q$ save del ruedo del sol $y$ de la luna y clip y estrellas y cometas, ora domingo y mes y año" (figg. 32-33).


Figure 33

M- That means Incas perfectly knew the cycles of Sun, Moon, stars, comets and used to easily predict eclipses of Sun and Moon. They also harmoniously rhythmed time on hour, 10-day week, month and year!
$\mathbf{N}$ - Now, if we give the right name to the days of the Inca 10 -day week, we can completely understand the Inca calendar.

M- Although Guaman Poma yupana generally reflects a diffuse venusian light, its five levels are resolutely related with Earth, Moon, Venus, Mercury and Sun (fig. 34). More in detail the first level, with its day-unit, is obviously coupled with the Earth (then the rotation of the Earth on its axis was well known by Incas!). The second level, with its twelve moons, is inevitably linked to the Moon, while the third, with its eight-year cycle, surely belongs to Venus. As concerns the 160year cycle Mercury is given the fourth level, while the precession of the equinoxes assigns the fifth to the Sun.


Figure 34
$\mathbf{N}$ - Then you suggest these Inca names for the 10-day week: 1) Earth-day, 2) Moon-day, 3) Venusday, 4) Mercury-day and 5) Sun-day ... Very, very interesting!

M- I cannot understand why only five days have a name ...
$\mathbf{N}$ - Don't you remember Tom Zuidema calendar? What's the distance between two consecutive solar-yellow diagonals (fig. 18)?

M- What an excellent remark! A gap of $\mathbf{5}$ days separates two consecutive Sun-days: so we have all the names, just to repeat for covering a whole 10-day week.
$\mathbf{N}-\ldots$ These days are like the fingers in a hand: we need to symmetrically repeat them in the other hand!

M- By the way! Two black seeds, both weighing 5, on the first row of Guaman Poma yupana confirm a 10-day week which is structured on two similar 5-day groups. All Andean calendars have a sublime architecture: its basic module is number $\mathbf{5}$, the number of celestial bodies that are inside the orbit of the Earth!
$\mathbf{N}$ - Incas not only achieved a perfect value of sidereal year by opportunely inserting a $\mathbf{5}$-day corrective, but trusted in an ingenious calendar, woven on simple canvas, whose structure had the rhythm of number 5 .

M- That's why we are so late in understanding their culture. To be honest it's very difficult to realize an equivalence between their little intelligent canvases and our silly tons of paper we waste in our calendars.
$\mathbf{N}$ - You are right! The Inca two-year calendar challenges millions, maybe billions years. It deserves a particular name: Perpetual Calendar! What's more we can get perfection in our calendar simply copying Inca system, without upsetting our loved 7-day weekly order.

M- Yes we can! Just using a 7-day corrective! It's enough to insert 7 days each 28-year cycle ( 20 x $7 / 5$ ), plus $\mathbf{7}$ days each 1,120-year cycle ( $800 \times 7 / 5$ ), plus 7 days each $\mathbf{6 2 , 7 2 0}$-year cycle ( $44,800 \times$ 7/5). It would be wonderful, because we'd have the same Inca accuracy and only one seven-year calendar, from Monday-year (which starts and ends on Monday) to Sunday-year (which starts and ends on Sunday), passing through Tuesday-year, Wednesday-year, Thursday-year, Friday-year and Saturday-year.
... Incas can still help us ... I am overcoming by wonder!
$\mathbf{N}-\mathrm{Me}$ too! It's better to stop now. We'll see tomorrow.
M- Bye, bye!
The day after ...
M- Someone is arriving ... Who is?
$\mathbf{N}$ - He's my friend Maurizio Orlando. Hi!
Orlando- Hi!
M- Hi!
$\mathbf{N}$ - What did you build up in two days time?
O- A prototype of an electronic calculator: its heart is perfectly Inca, because it works on the mixed base 36-40. I have followed your suggestion, adding three rows to Guaman Poma yupana; I think its best name is Atahualpa.

M- Atahualpa? ... Why not Guaman Poma? After all he surely is the savior of the Inca empire, perhaps of the human one!
$\mathbf{N}$ - There's a loved nephew! Mauro, Atahualpa is the last Inca emperor: an emperor is more representative of an entire culture than anyone else, even if genius...

O- ... We'd like that history restarted where it was ruthlessly interrupted ... Let's see its working in the four basic arithmetic operations. First we need to put on the necessary leds for the first number, then act the enter key, next put on the leds for the second number, finally activate the desired operation key and read the result in the leds.
E. $\mathrm{g} . \mathbf{3 6 \times 8 0} \mathbf{~} \mathbf{2 8 8 0}$ (fig. 34) with $\mathbf{8 0}=\mathbf{7 2 + 5 + 3}$.


Figure 34
$\mathbf{N}-\ldots$ Wahoo! ... Perfect in its own!
M- Atahualpa is the eighth wonder of the world! I wonder what Incas would have achieved without Spanish.

N- Why? Do you really think that Italians would have been better? Are you forgetting Cain? ... Human gender is sometimes violent!

In another room the kids Salvatore, Federico and Lorenzo are playing guitars, singing a beautiful universal dream song...

S, F, L-"Imagine all the people ..."

## ALL TOGETHER- ... "Living life in peace ..."

I feel as my duty to underline that this study is a new elaboration, in the form of a dialogue, of a single part belonging to a complex study about Andean cultures, dating back to 2004.
Unfortunately, neither my determined efforts nor my friends from all over the world could help me to find a suitable publisher.
I therefore apologize with You all -scholars, passionate researchers and friends- for the simple format I used, the only one I could afford.
Also my style has been accurately simplified so that this study could be more easily spread throughout the Internet.
In case any publisher preferably from south America were interested, he could contact me directly at Istituto di Istruzione Superiore "Alessandro Volta", Pescara.

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