Decimal Guaman Poma

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ABSTRACT. A 16th century chronicler makes a list of Incan numbers in his manuscript. Some terms including *huno*, almost impossible to translate, led scholars towards two distinct problematical solutions. Thanks to the help deriving from the redevelopment of the Incan calculating mechanism Mathematics is allowing us to get to the final solution of this huge linguistic problem. A better understanding of Incan numbers and their way to perform calculations will lead to further insights about Andean civilizations.

Double misunderstanding

Page 361 of the manuscript *"Nueva corónica y buen gobierno"*, written by Felipe Guaman Poma de Ayala (1613), is largely spent on Incan numbers: *"y contava desta manera comensando de uno dos y tres* (they counted in this way starting with one, two and three) *suc – yscay – quinza – taua – pichica – zocta – canchis – puzac – yscon – chunga – yscaychunga – quinzachunga – tauachonga – piscachunga – zoctachunga – canchischunga pozacchunga – ysconchunga – pachaca – uaranga – chungauaranga – huno – pachacahuno – uarangahuno – pantacachuno* ————*"*.

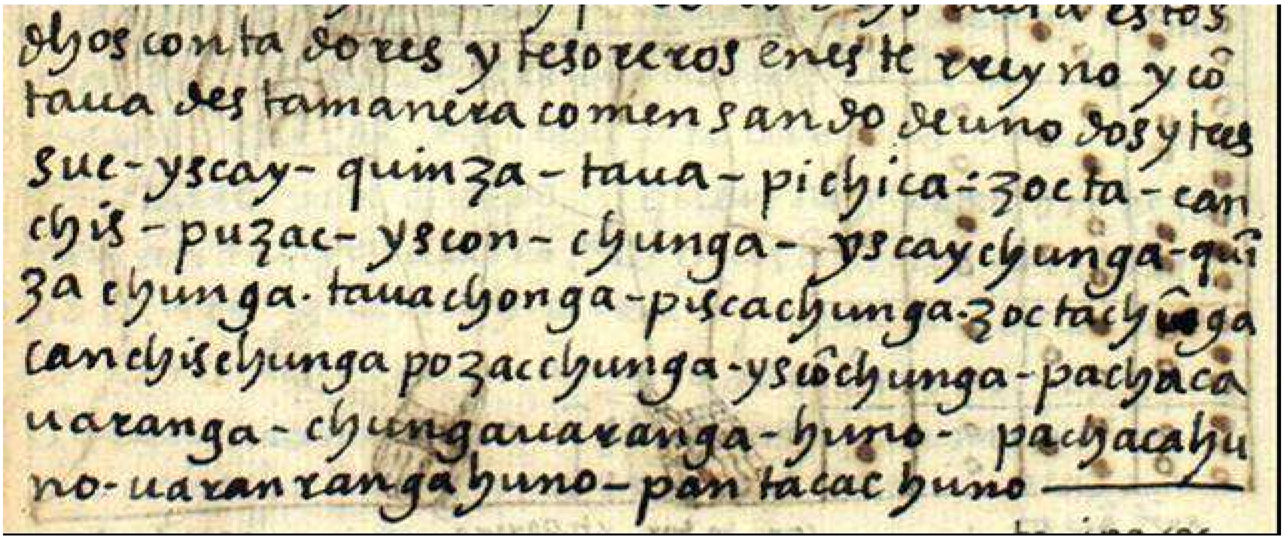
dyos conta dores y tesoreros en este reyno y cō
taua desta manera comensando de uno dos y tres
suc - yscay - quinza - taua - pichica - zocta - can
chis - puzac - yscon - chunga - yscaychunga - qui
za chunga. tauachonga - piscachunga. zoctachunga
canchischunga pozacchunga - yscōchunga - pachaca
uaranga - chungauaranga - huno - pachacahu
no - uaranrangahuno - pantacachuno

Figure 1

In this page (fig. 1) there are lots of dashes not having their typical punctuation mark role. They actually distinguish the numbers from one another; the same page has been till now considered deeply dark, because it generates an interpretative problem; so scholars have been pushed toward two solutions, very different from each other. Unfortunately both solutions have proved to be wrong, because they still contain unsolvable questions.

If we consider the detailed sequence from *suc* to *chunga* it is impossible to have controversies; the sequence of ones *suc* (**1**), *yscay* (**2**), *quinza* (**3**), *taua* (**4**), *pichica* (**5**), *zocta* (**6**), *canchis* (**7**), *puzac* (**8**), *yscon* (**9**), is rhythmed by the accurate opening direction *"comensando de uno dos y tres"* (starting with one, two and three); the sequence of tens *chunga* (**10**), *yscaychunga* (**20**), *quinzachunga* (**30**), *tauachonga* (**40**), *piscachunga* (**50**), *zoctachunga* (**60**), *canchischunga* (**70**), *pozacchunga* (**80**), *ysconchunga* (**90**), obtained by simple combinations (e.g. *canchischunga*–**70** derives from *canchis*–**7** and *chunga*–**10**), unanimously links number **100** to *pachaca*, making the sequence of hundreds unnecessary (e.g. *tauapachaca* is **400**). Just the lack of hundreds **200**, **300**, …., **900** allows Guaman Poma to insert the exponential sequence *pachaca* (**100**), *uaranga* (**1,000**), *chungauaranga* (**10,000**), with the self-evident correlations *uaranga*–**1,000** and *chungauaranga*–**10,000**, (a combination between *chunga*–**10** and *uaranga*–**1,000**).

The interpretative division regards the remaining numbers including *huno*.

According to some scholars *huno* would be **10,000**. As a matter of fact they somehow give their own interpretation forcing the original text "*chungauaranga – huno*" to become "*chungauaranga, es un* (is a) *huno*", overflying on the dividing role of the dash. With this behavior a little autoattractive and more than a little arbitrary, they would have the correlations *pachacahuno*–**1,000,000** (**100**–*pachaca* x **10,000**–*huno* = **1,000,000**), *uarangahuno*–**10,000,000** (**1,000**–*uaranga* x **10,000**–*huno* = **10,000,000**), while the eccentric *pantacachuno* would be the equivalent of infinity. As a consequence they obtain a strange enough exponential sequence: *suc* (**1**), *chunga* (**10**), *pachaca* (**100**), *uaranga* (**1,000**), *chungauaranga* (**10,000**), *huno* (**10,000**), ?, *pachacahuno* (**1,000,000**), *uarangahuno* (**10,000,000**), …?, *pantacachuno* (infinity), with the unnecessary repetition of **10,000** (which would be named in two different ways *chungauaranga* and *huno*!), the unexplainable lack of **100,000** and the unfounded jump in the darkness from **10,000,000** to infinity; these three shortcomings are well highlighted in the second column of Figure 2.

	huno=10,000	**huno=1,000,000**
pantacachuno	**infinity?**	**infinity?**
uarangahuno	**10,000,000?**	**1000,000,000?**
pachacahuno	**1,000,000?**	**100,000,000?**
huno	**10,000?**	**1,000,000?**
chungauaranga	**10,000**	**10,000**
uaranga	**1,000**	**1,000**
pachaca	**100**	**100**
chunga	**10**	**10**
suc	**1**	**1**

Figure 2

Other scholars, who might have trusted Gonzales de Holguín, who translates *hunu* into *un millon* (one million) in his *Vocabulario* (Holguín 1608), say that *huno* would be **1,000,000**; with this really neocolonialistical approach they would have the correlations *pachacahuno*–**100,000,000** (**100**–*pachaca* x **1,000,000**–*huno* = **100,000,000**), *uarangahuno*–**1,000,000,000** (**1.000**–*uaranga* x **1.000.000**–*huno* = **1,000,000,000**) and the extroverted *pantacachuno* with the same meaning of infinity, notably after the fact that the same *Vocabulario* translates *Panta cak hunu* into *Innumerable o numero infinito que desatina al contarlo*. Following this approach (third column of fig. 2) they surely face a triple fracture in the exponential sequence: *suc* (**1**), *chunga* (**10**), *pachaca* (**100**), *uaranga* (**1,000**), *chungauaranga* (**10,000**), ?, *huno* (**1,000,000**), ?, *pachacahuno* (**100,000,000**), *uarangahuno* (**1,000,000,000**), …?, *pantacachuno* (infinity), lacking **100,000**, **10,000,000** and the entire series which fills the chasm between **1,000,000,000** and infinity.

So both solutions, summarized in Figure 2, are wrong with regard to the numbers containing *huno*; in the exponential sequence, so well delineated by Guaman Poma (**1**, **10**, **100**, **1,000**, **10,000**), the lack or repetition of some terms hasn't got any mathematical sense and neither zero nor infinity can be represented, unless the sequence becomes a series, that's to say an entity with infinite terms.

Such a circumstance has to be resolutely excluded in relation to the finite list on page 361 we are examining. In addition to that, lacking zero in the sequence, infinity (its inverse) absolutely must lack; at least one thing remains certain: ***pantacachuno* can not be infinity**!

Figure 3

The transferring of our idea of infinity to Guaman Poma would certainly cause a violent conflict between our world and his universe, always synthesized by the poetic power of children. Who could determine, for instance, the size of *Castilla* and *Pirù* while observing the sketch *Pontifical Mundo* (fig. 3) on page 42? At most, if we considered the deep meaning of high-low polarity in Andean cultures[1] and the location of *Pirù* just below the sun, we could see a superiority of the Incan civilization. Such a lyrical approach couldn’t objectively include the concept of infinity!

[1] This theme has been deeply investigated by Liliana Rosati (2002).

Then how could we go on?

The exponential sequence …, *suc* (**1**), *chunga* (**10**), *pachaca* (**100**), *uaranga* (**1,000**), *chungauaranga* (**10,000**),…, allows, on the right, the problematic insertion of terms 100,000 (*pachacauaranga*–100.000 is absent in the list on page 361), 1,000,000, etc .. and, on the left, the insertion of terms **1/10**, **1/100**, **1/1,000**, etc … Just this last chance, hardly conceivable before decoding the Incan calculating device, is fundamental in determining the meaning of the terms containing *huno*.

Exponential Contiguity

The same *"Nueva corónica"* offers on page 360 a wonderful sketch (fig. 4), representing the powerful Incan counting device, a matrix with five rows and four columns working on an advanced biplace notation system. Just this device is the key to decode the great mathematical and astronomical Incan knowledge.

Figure 4

If we give the weights **1**, **2**, **3** and **5** to the kernels we discover a fully-developed numeral system in the mixed base **36/40** (De Pasquale 2011).

Using a reduced number of kernels –i.e. two with value **3**, one with value **2**, one with value **1** (and two valuing **5** for carrying management)–, we can work with our favourite decimal system (**2x3 + 2 + 1 = 9**) which, according to scholars, is the base of *quipus* records and Incan societal organization.

Then in this magnificent civilization two numeral systems lived together in perfect harmony, one for astronomical purposes, another for ordinary counting.

If we want to perform the addition **7** + **9** in base ten (Figure 5) we put one **5**-value and one **2**-value kernels (**5** + **2** = **7**, fig. 5a) and after one **5**-value, one **3**-value and one **1**-value kernels (**5** + **3** + **1** = **9**, fig. 5b), obtaining the brute sum (fig. 5c); two quintets (fig. 5d) snap the carry (fig. 5e); it is possible to have the last quintet (**3** + **2**= **5**, fig. 5e), and read the result **16** (**1** ten plus **6** ones).

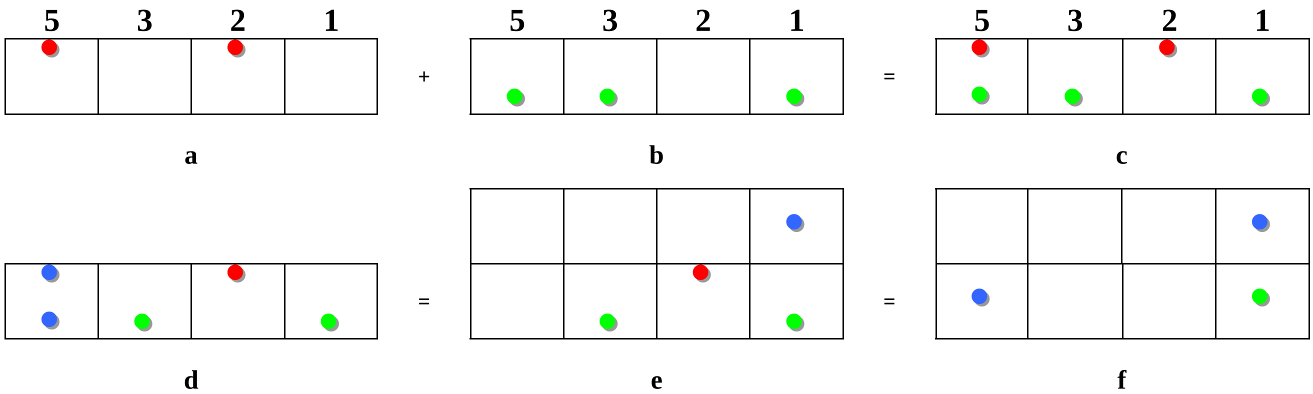

Figure 5

But let's get back to page 361 (fig. 1)! Guaman Poma writes about "*dhos contadores y tesoreros*", two counters and treasurers, working in the societal structure, clearly alluding to the most common numeral system in base **10**.

If we use his matrix in base ten the rows become those represented in Figure 6 (carries are dark grey backcolored), with the interruption of the sequence of direct numbers at the order of tenthousand, because the matrix has only five rows.

(2x50,000)	**2x30,000**	**20,000**	**10,000**	***(chungauaranga)***
(2x5,000)	**2x3,000**	**2,000**	**1,000**	***(uaranga)***
(2x500)	**2x300**	**200**	**100**	***(pachaca)***
(2x50)	**2x30**	**20**	**10**	***(chunga)***
(2x5)	**2x3**	**2**	**1**	***(suc)***

Figure 6

While considering the inverse numbers, on the same matrix, we have the situation illustrated in Figure 7, with the interruption at the order of tenthousandth.

(2x5)	**2x3**	**2**	**1**
(2x0.5)	**2x0.3**	**0.2**	**0.1**
(2x0.05)	**2x0.03**	**0.02**	**0.01**
(2x0.005)	**2x0.003**	**0.002**	**0.001**
(2x0.0005)	**2x0.0003**	**0.0002**	**0.0001**

Figure 7

The **<u>true correlations</u>** come out by their own:

***suc* = 1**
***huno* = 1/10**
***pachacahuno* = 1/100**
***uarangahuno* = 1/1,000**
***pantacachuno* = 1/10,000**

with *huno* mathematically meaning "decimal". Such correlations even explain the lack of *chungahuno* in the numerical list on page 361.

One tenth, the decimal of the first order, is the first decimal and can be identified only with *huno*; *pachacahuno* and *uarangahuno* show an unassailable coherency with **1/100** (*pachaca*-**100** and *huno*-decimal) and **1/1,000** (*uaranga*-**1,000** and *huno*-decimal)[2], while *pantacachuno* (**1/10,000**) represents an irregularity which highlights the smallest value in Incan societal organization (we should not forget that family units used to be gathered up to **10,000** people; so **0.0001** means a person in the biggest social group). Even in the extraction of square roots, the Incan daily devices, like mountains with seven terracing, allow accurate performances till to the third decimal, confirming the exceptionality of the tenthousandths (*pantacachuno*) in ordinary counting[3].

chungauaranga	**10,000**
uaranga	**1,000**
pachaca	**100**
chunga	**10**
suc	**1**
huno	**0.1**
pachacahuno	**0.01**
uarangahuno	**0.001**
pantacachuno	**0.0001**

Figure 8

Figure 8 shows the strong cogency of this tardy solution, with the exponential sequence ***pantacachuno* (1/10,000), *uarangahuno* (1/1,000), *pachacahuno* (1/100), *huno* (1/10), *suc* (1), *chunga* (10), *pachaca* (100), *uaranga* (1,000), *chungauaranga* (10,000)** having a central symmetry with regard to the one ***suc*** and, most of all, **with no repetition or lack**.

The same sequence develops like the musical octaves in purity of resonance, that's to say in eight intervals of exponential harmonic contiguity.

Mathematical Truth

The hypothesis that the terms *pantacachuno* through *huno* are representative of decimal numbers is authoritatively supported by mathematical logic as well as by some corollary elements, absolutely not negligible. Let's consider them in detail:

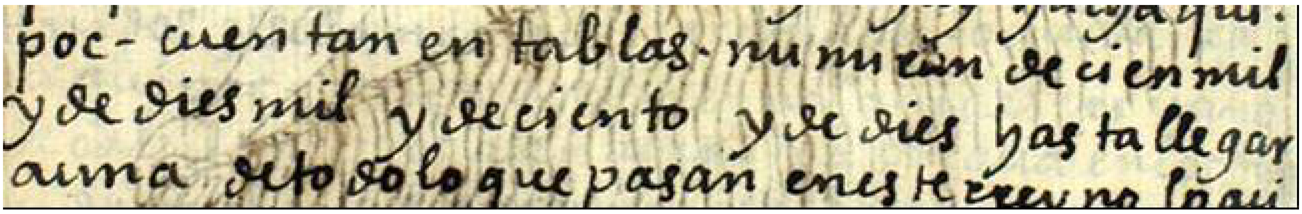
poc- cuentan en tablas. numincan de cien mil
y de dies mil y deciento y de dies hasta llegar
auna dito solo que pasan en este reyno

Figure 9

[2] As if we named 0.01 *hundred decimal* and 0.001 *thousand decimal*.

[3] The extractor of square roots is very interesting and it will be subject of another investigation.

- Guaman Poma, with his statement reported on page 361 (fig. 9) *"numiran de cienmil y de diesmil y de ciento y de dies hasta llegar a una"* (they number 100,000 and 10,000, and 100 and 10 until arriving at **una**), ascribs number 100,000 to Incan ordinary counting[4], and beside that inserts an exact ratio, based on 1,000 as divisor, which directly leads to decimals; in fact:

  100,000/10,000 (*cienmil y diesmil*) **= 100/10** (*ciento y dies*)**= 0.1/0.01** (***una***!)**,**
  (**100,000/**1,000**=100** and **10,000/**1.000**=10** and likewise **100/**1,000**=0.1** e **10/**1,000**=0.01**); then the word ***"una"***, phonetically analogous to *huno*, has logical ties with **0.1** and **0.01**, the decimals of the first and second order. This directly implies that ***una*** means "decimal set or family" and not "one" as translated to date.

- The role of *huno apo*, inserted by Guaman Poma on page 65, is more prestigious than that of *guaranga curaca* translated as *señor de mil yndios* (Lord of one thousand Indians); *huno apo*, described as *señor de dies mil yndios* (Lord of tenthousand Indians), actually has the suggestive meaning of "**Lord of tenth**", because ten *guaranga curacas* (local officers), each one representing one tenth, were responsible for their actions to a single *huno apo* (supervisor of ten *guaranga curacas*).

- Gonzales de Holguín, caused quite a big confucion, with his *Vocabulario* which translates *Hunu* into *Un millon* (one million). But he enters in a complete contradiction with his own while translating ***Hun****tta – Lleno…en cosas liquidas* (full with liquids), and *Tassacta* ***hun****ttani – Pagar cumplida la tassa* (to fully pay taxes) Well, how could we fill any container to a million and not to the last drop, which clearly is a decimal? And how could we pay taxes to a million and not to little decimal fractions[5]?

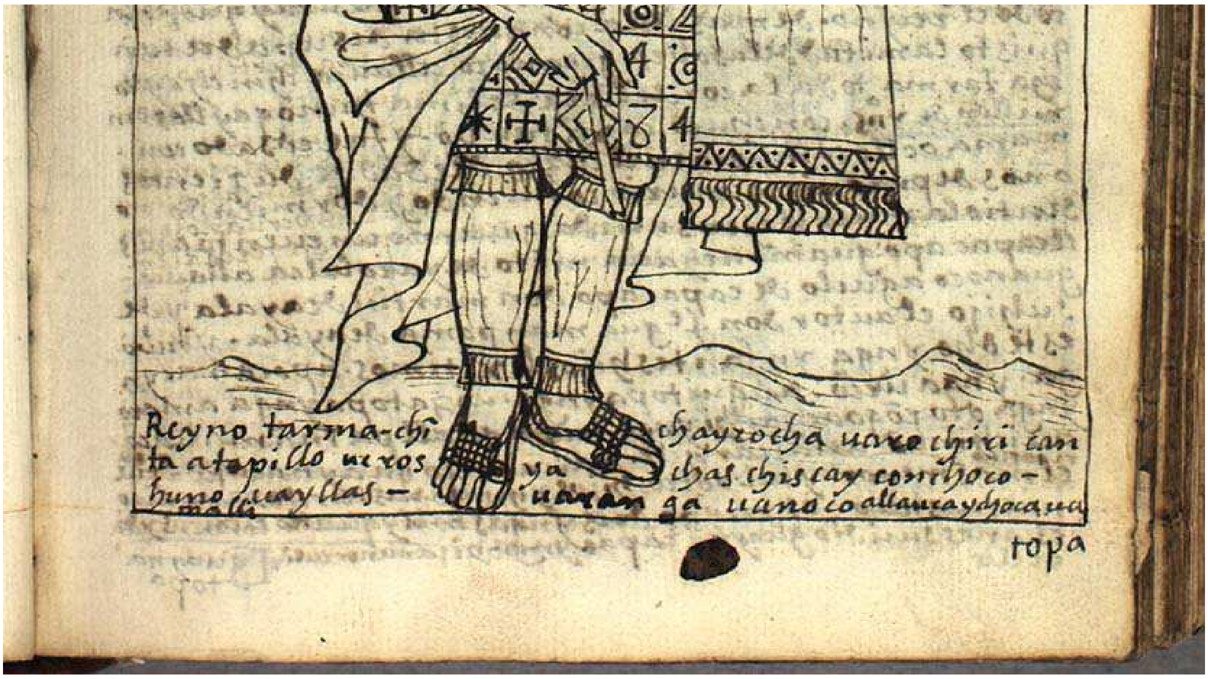

Figure 10

- In the list of provinces which the Inca *Topa Iupanqui* reigned over, on page 110 (fig.10), Guaman Poma put ***huno*** *uayllas* at the **tenth** place (in good evidence, between two dashes), with the unmistakable meaning of **tenth** province; and if we read on page 111 (fig. 11) "*huno*

[4] When fulfilled in base ten his matrix is representative of 99,999. In the ancient quechua 100,000 is *pachacauaranga*.
[5] Exchanging one tenth with one million, also inadvertently, it means eliminating the calculation subtlety related to decimals, dramatically diminishing Incan civilization.

gayllas, un millon de yndios" (one million Indians), one million, in full disagreement with Gonzales de Holguín and his epigones, doesn't refer to *huno*! The **tenth** province ***huno*** *gayllas* and other nine provinces, put together, had one million Indians. As a consequence we can know a precious, fundamental datum: each province was generally made up of 100,000 Indians (with 100 *guaranga curacas* and 10 *huno apos*)!

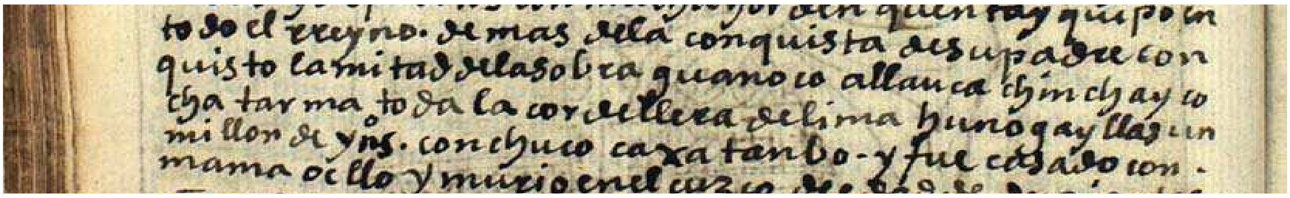

Figure 11

- The dash beyond ***pantacachuno*** is much longer than the other marks (fig. 12), as if Guaman Poma wanted to stress the need of stopping. So ***pantacachuno*** is the smallest decimal in Incan counting system!

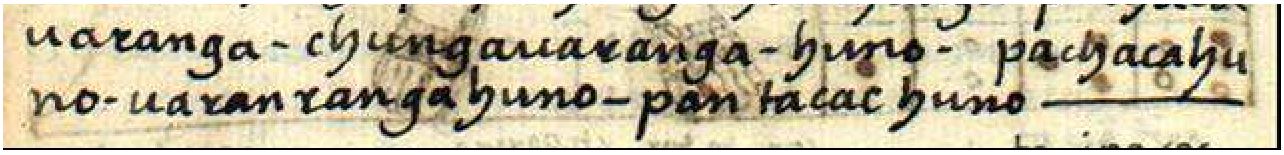

Figure 12

- According to Liliana Rosati the philological analysis (specifically made for us) excludes that ***pantacachuno*** could be infinity, because the term itself suggests a feeling of loss linked to smallness.

- The Incan ten-day week, defined ***hun****ca* ***hun****ac* by Guaman Poma on page 884, clearly has the same root of ***hun****o*; it has the fascinating meaning "from tenth to tenth". Such a philological datum definitely confirms that neither 10,000 nor 1,000,000 can be in equivalence with huno.

These correlations, so hardly obtained, on one hand surely pose serious problems on reliability of many colonial texts, which try to flatten the Andean peaks of Incan culture, with the result of safe guarding, even at an unconscious level, a hypothetical primacy of the invader. On the other hand they show an attitude of strict self-censorship by indigenous and mestizo chroniclers, quite evident in the "*Nueva corónica*", with Guaman Poma not mentioning any use of the matrix drawn with his own hands on page 360!!! These chroniclers, particularly aware of the Incan greatness, deliberately use initiation cryptic strategies, obscure for everyone having an attitude of superiority. In order to preserve their cultural heritage, they prefer to wait over the centuries for some divinely inspired dowser, able to decode their precious secrets. In this way they were able to make fun of the double censor!

Now if the neocolonialistical approach, used to date, is realistically shallow is it the case of surrendering to discouragement, because of the huge work that awaits us in the reinterpretation of a whole literature?

Isn't perhaps Mathematics a foolproof method, even capable of solving linguistic problems, incontaminable by all sorts of prejudices and cryptic formulae? Isn't Mathematics able to always show Truth thanks to its intrinsic and universal beauty, even after an inexplicable delay of five centuries?

Harmonized numbers

Subtraction is an amusing computation. If we look at the case **815** – **263** the minuend (**815** fig. 13a) is harmonized on the subtrahend (fig 13b), that's to say that it must clearly contain the smaller number **263** (fig. 13c).

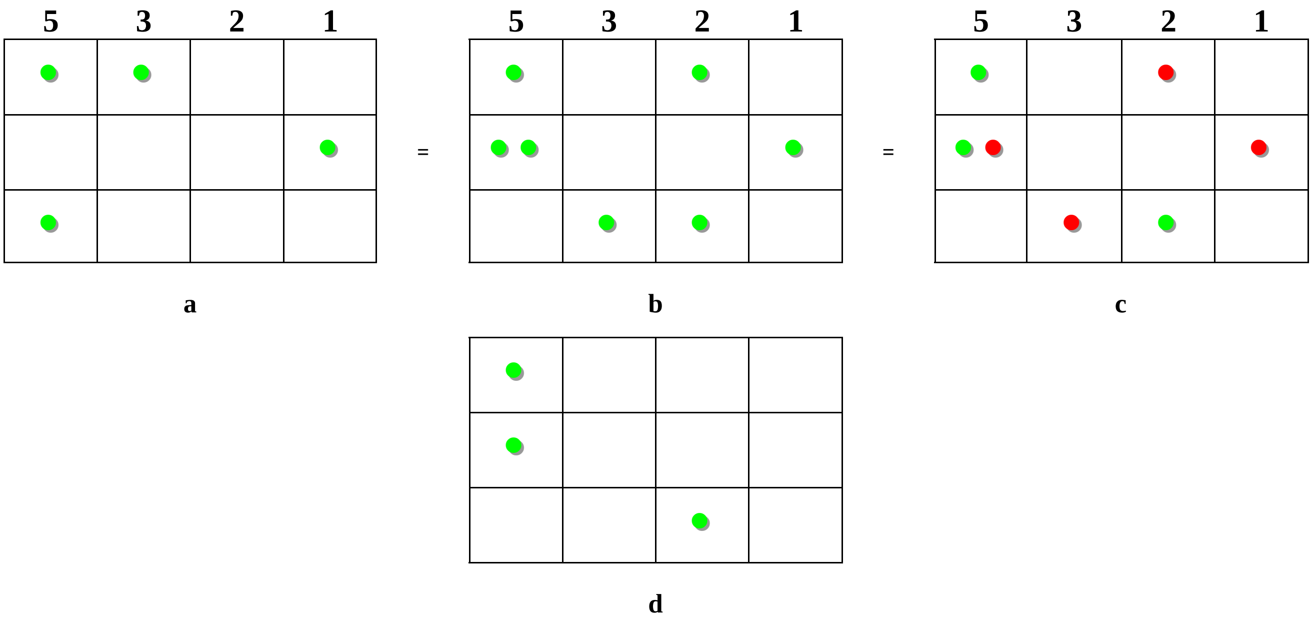

Figure 13

After that we cut away, by a real subtraction, the kernels representing **263**. The number that remains on the matrix is simply the result of the subtraction, this time being **552** (fig. 13d)[6]. Maurizio Orlando has been the first to propose the harmonization method (Orlando 2004).

The Reducer (Fibonacci)

According to Mariarita Laganà Incans used to work with a reduced Pythagorean table, being 5x5 = 25 the most complicated multiplication (Laganà 2004), because of the structure of their abacus on Fibonacci sequence. Let's perform **46 x 152**, with **152** broken down into the sum **2 + 50 + 100** which is easier to work with.

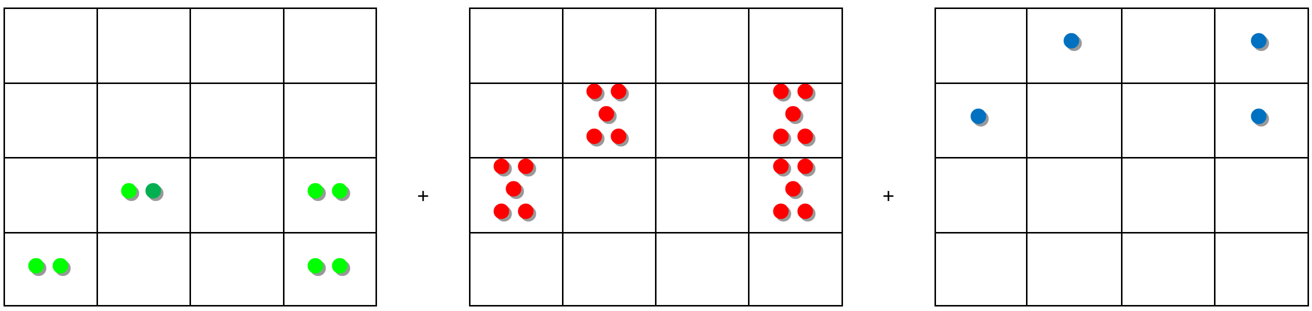

Figure 14

So **46 x 2** is obtained by simply doubling the kernels related to **46** (fig. 14a); **46 x 50** (fig 14b) is a combination of the shift of one row above (x 10) and the quintupling of kernels (x 5); **46 x 100** (fig. 14c) requires the **46**-value kernels to be shifted of two rows above.

Adding together these partial products (fig. 15a,b,c) we have the product of the multiplication **46 x 152**, in this case being **6992** (**92** + **2300** + **4600** = **6992** fig 15d).

[6] It is also possible to perform subtractions in a canonical way, i.e. starting from ones.

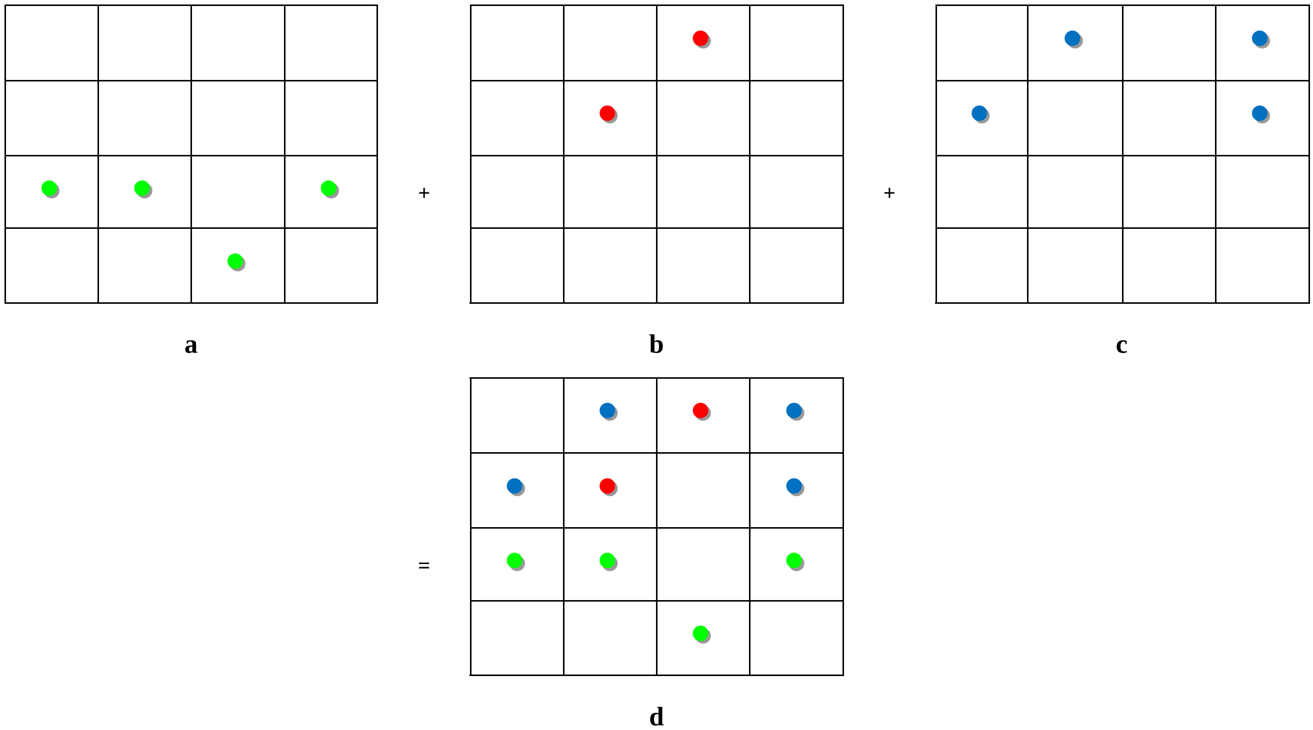

Figure 15

Harmonizing to the end

Division is the most amusing operation, generally requiring a multiharmonization of the dividend on the divisor. Let's look at the problem **6992**/**152** **(**fig. 16a). In the first step we dispose **6992** as **4 x 1520** with the remainder **912** (fig. 16b). It's easily observable that (**4 x 1520**)/**152** = **40** (fig.16c). In the second step the remainder has to be harmonized on the divisor (fig. 16d) as **6 x 152** (fig. 17a), with **(6 x 152)/152** = **6**. At last we have the quotient, in our case **40 + 6 = 46** (fig.17b): it's really amazing!

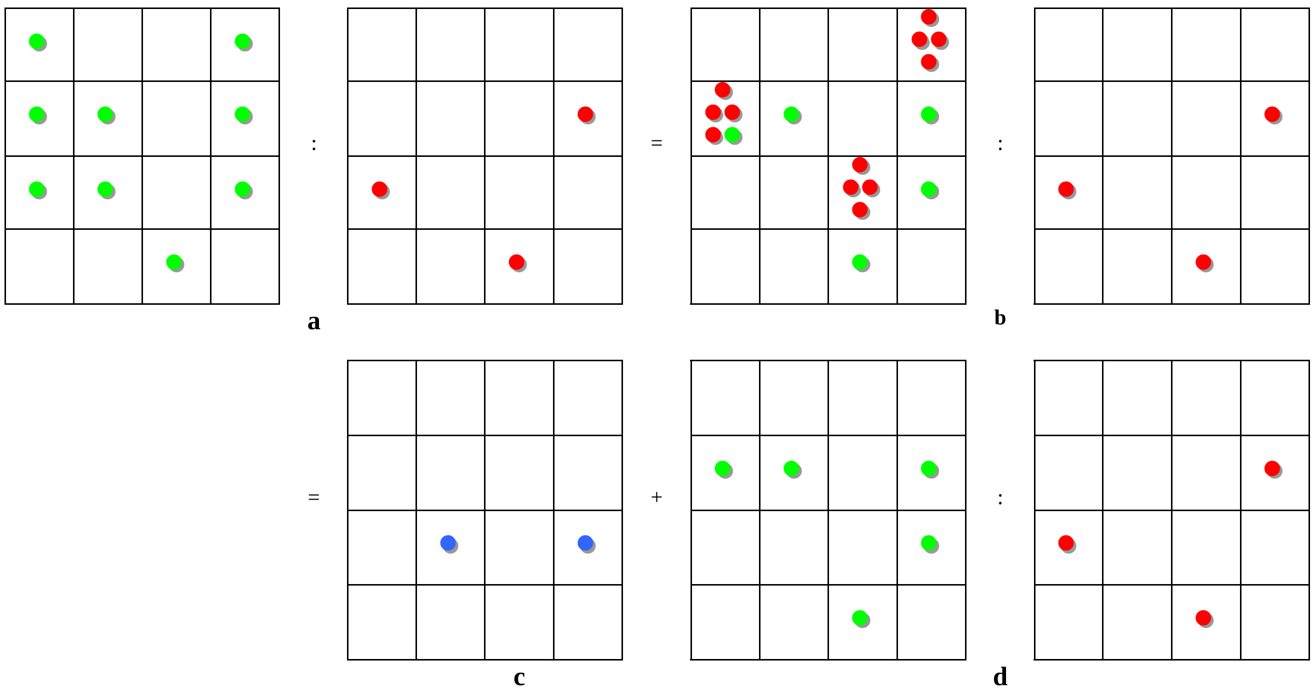

Figure 16

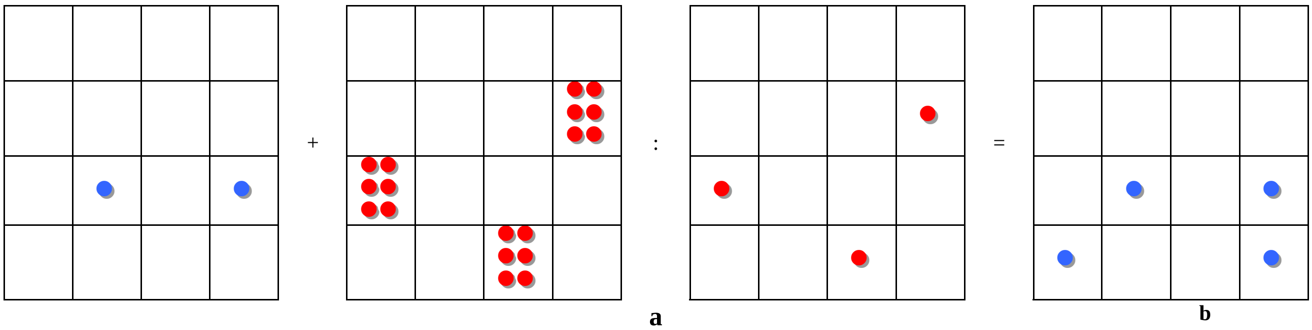

Figure 17

How many bases?

Incan abacus is a very flexible counting mechanism. It can indifferently work not only in base **10** and **36/40** but also in the bases **12**, **13**, **18**, **20**, etc .. It depends on the kernels per row. Figure 18 shows some details.

base	5	3	2	1
10		●●	●	●
	●	●		●
12	●	●	●	●
13		●●	●●	●●
18	●●	●●		●
20	●●	●●	●	●
36	●● ● ●●	● ●●		●
40	●● ● ●●	● ●●	●●	●

Figure 18

The possibility of working in a mixed base **18/20** is particularly interesting because it could explain cultural ties and exchanges of astronomical data with Mesoamerican civilizations.

Conclusion

Guaman Poma is the Inca of dissemblers. He worked very hard pretending to extirpate local idols in order to get the permission of writing the “*Nueva Coronica*”. Always faithful to his people, he came up with obscure solutions which, after five centuries, have the irresistible power of Truth.

Thanks to his efforts we are getting to a better knowledge of Incan civilization.

How could we show him our deep gratefulness?

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